[This question paper contains 4 printed pages.]

Your Roll No.....

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Sr. No. of Question Paper : 1666

Unique Paper Code : 42357618

Name of the Paper : DSE – NUMERICAL METHODS

Name of the Course : **B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. **All** questions carry equal marks.

Q 1.(a) If X = 2.536, find the absolute error and relative error when X is rounded-off to two decimal digits. (6.25)

(b) Find the relative error of the number 9.5, if both of its digits are correct. (6.25)

(c) Write the order of convergence of Bisection Method, Secant Method and Newton-Raphson Method. Also name these methods in decreasing order w.r.t. rate of convergence. (6.25)

P.T.O.

(d) Determine the number of significant digits in the following numbers. (6.25)

(I) 0.60549, (II) 8.00889, (III) 458900, (IV) 0.87×10^{-9}.

Q. 2(a) Perform five iterations of the Bisection method to obtain the smallest positive root of the equation $f(x) = x^3 + 4x - 1 = 0$. (6.25)

(b) Using Regula-Falsi method compute the real root of the equation $x^2 = 3$. Correct up to four decimal places. (6.25)

(c) Using Newton-Raphson Method compute $\sqrt{19}$ correct to four decimal places. (6.25)

(d) Using Secant method find the smallest positive root of the equation $x^3 - 2x^2 = 2$ correct up to three decimal digits. (6.25)

Q. 3 (a) Explain the term partial pivoting. Solve the following system of equations using Gauss-Jordan method (6.25)

$$2x+6y+10z = 1,$$
$$x+3y+33z = 2,$$
$$3x+14y+28z = 33,$$

(b) Approximate the solution of AX= b where (6.25)

$$A = \begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix}$$

with $x^{(0)} = [0,0,0]^{(t)}$ using Gauss-Seidel iteration method by performing three iterations.

(c) Consider the following table: (6.25)

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Obtain the Newton forward and Newton backward difference polynomials. Are they same? Estimate f(0.35).

(d) Obtain the piecewise quadratic interpolating polynomial for (6.25)

x	-2	-1	1	2	4
$f(x)$	-29	-8	-2	-5	7

Interpolate at x = 3.0.

Q. 4 (a) Perform three iterations to solve the linear system (6.25)

$$2x - y + z = -1,$$
$$x + 2y - z = 6,$$
$$x - y + 2z = -3,$$

using Gauss-Jacobi iteration method by taking the initial approximation as $(x,y,z) = (0,0,0)$.

(b) Construct the interpolating polynomial by using Gregory- Newton backward difference interpolation formula for the given data: (6.25)

x	1	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3891	12.1825

Estimate the value of f(2.25).

(c) Show that (6.25)

(i) $\mu = \left[1 + \frac{\delta^2}{4}\right]^{1/2}$, (ii) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$.

Also, if $f(x) = \frac{1}{x^2}$, find the divided difference $f[x_1, x_2, x_3, x_4]$.

(d) Given that f(0)= 1, f(1)= 3, f(3)= 55, find the unique polynomial of degree 2 or less, which fits the given data by Lagrange interpolation. (6.25)

Q. 5(a) Compute the value of $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule taking n = 6. (6.25)

(b) Find Richardson extrapolation of $f(x) = -2e^{-2x}$ when x=0.35, h=0.25 with the help of central divided difference formula $f'(x) = \frac{f(x+h) - f(x-h)}{2h}$. (6.25)

(c) Compute the value of $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's one third rule. (6.25)

P.T.O.

(d) Evaluate $\int_1^7 \frac{dx}{x}$ by Richardson's extrapolation method using central divided difference formula $f'(x)=\frac{f(x+h)-f(x-h)}{2h}$. (6.25)

Q. 6(a) Calculate by Simpson's rule an approximate value of $\int_{-3}^{3} x^4\,dx$ by taking seven equidistant ordinates. (6.25)

(b) Solve by Euler's method, the initial value problem (6.25)

$\frac{dy}{dx}=\frac{x-y}{2}, y(0)=1$ over [0, 3], using step size 0.5.

(c) Apply Heun's method to compute $y(0.2)$ where $\frac{dy}{dx}=x+2y,\ y(0)=0,\ h=0.$ (6.25)

(d) Compute $\int_1^6 x^3\,dx$ by trapezoidal rule with n = 4. (6.25)